Chapter 7 Review Sheet, Covers 7.1-7.7
Math 280, Vanden Eynden

1. $\int_0^1 x^2 e^{2x}\, dx$

2. $\int \sin^3\theta \cos^7\theta \quad d\theta$

3. $\int \tan(4x)\sec^4(4x) \quad dx$

4. $\int \frac{x^2}{x^2 - 3x + 2}\, dx$

5. $\int \frac{7x-2}{x^3 - 2x^2}\, dx$

6. $\int \sin 4t \cos 3t \quad dt$

7. $\int 3x^5 \ln(2x) dx$

8. $\int 5^x \sin x\, dx$

9. $\int x \csc x \cot x \quad dx$

10. $\int \frac{x^2}{\sqrt{5 + x^2}} \quad dx$

11. $\int \frac{dx}{\left(4x^2 - 9\right)^{5/2}}$

12. $\int \sin^2\theta \cos^2\theta \ d\theta$

13. $\int \sqrt{x} e^{\sqrt{x}} dx$

14. $\int x\sqrt{6x - x^2 - 8} \quad dx$

15. Estimate the value of $\int_1^4 \sqrt{x}e^{\sqrt{x}}\,dx$ using:

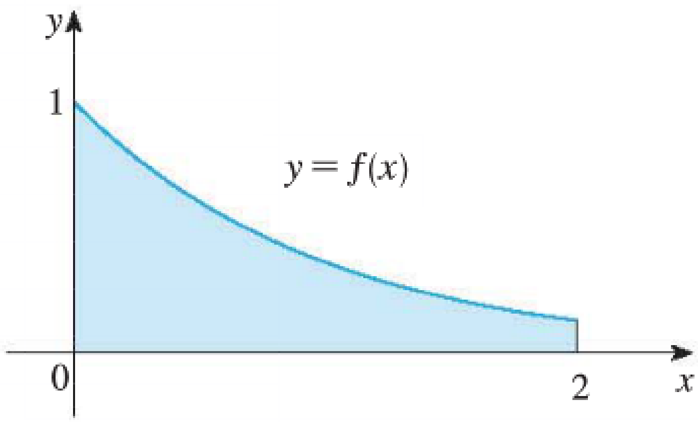
(a) Midpoint, (b) Trapezoidal, and (c) Simpson's Rule with n=6

16. The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate $\int_0^2 f(x)dx$, where f is the function whose graph is shown below.

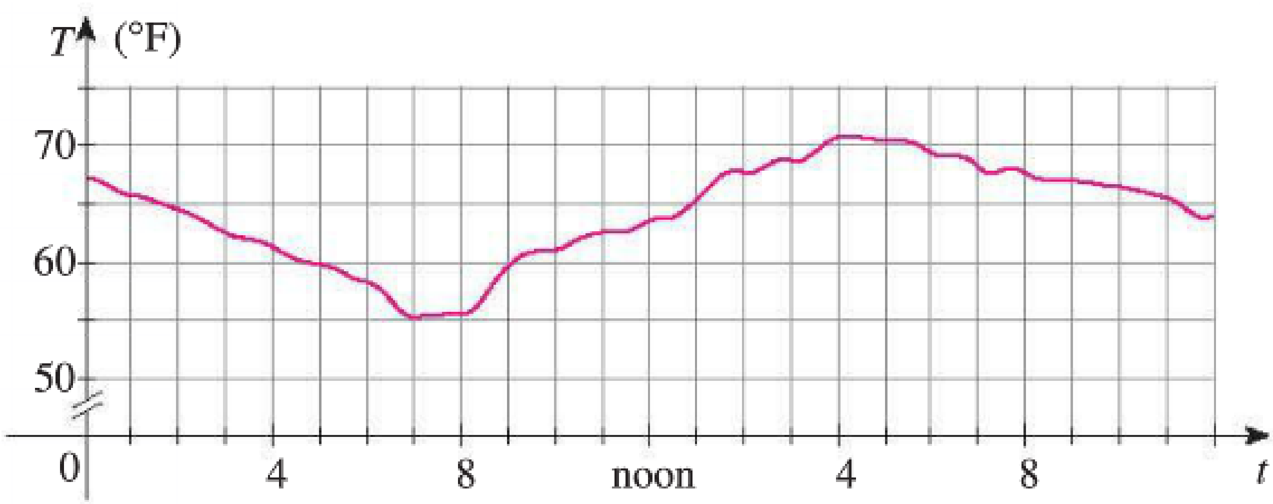
The estimates (listed in increasing order) were 0.7811, 0.8632, 0.8675, and 0.9540, and the same number of subintervals were used in each case.

(a) Which rule produced which estimate?

(b) Between which two approximations does the true value of $\int_0^2 f(x)dx$ lie?



17. A graph of the temperature in New York City on 9/19/09 is shown. Use Simpson's rule with n=12 to estimate the average temperature on that day.



Chapter 7 Exam Formula Sheet (You will be provided a fresh copy on exam day)

Math 280, Vanden Eynden

Derivatives of Inverse Trig Functions:

$$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}x=\frac{-1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$$

Established Integration Formulas

$$\int \tan x\,dx=\ln|\sec x|+C \qquad \int \sec x\,dx=\ln|\sec x+\tan x|+C$$

$$\int \cot x\,dx=\ln|\sin x|+C \qquad \int \csc x\,dx=\ln|\csc x-\cot x|+C$$

Half-Angle Formulas

$$\sin^2 x=\frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x=\frac{1}{2}(1+\cos 2x)$$

$$\tan^2 x=\frac{1-\cos 2x}{1+\cos 2x}$$

Double-Angle Formulas

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Product Formulas

$$\sin A\cos B=\frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$$

$$\sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

ANSWERS to Chapter 7 Review Sheet, Covers 7.1-7.7
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1. $\frac{1}{4}e^2 - \frac{1}{4}$
2. $-\frac{\cos^8\theta}{8} + \frac{\cos^{10}\theta}{10} + C$ or $\frac{\sin^4\theta}{4} - \frac{\sin^6\theta}{2} + \frac{3\sin^8\theta}{8} - \frac{\sin^{10}\theta}{10} + C$
3. $\frac{1}{16}\sec^4(4x) + C$
4. $x + 4\ln|x-2| - \ln|x-1| + C$
5. $-3\ln|x| - \frac{1}{x} + 3\ln|x-2| + C$
6. $-\frac{1}{2}\cos t - \frac{1}{14}\cos(7t) + C$
7. $\frac{1}{2}x^6\ln(2x) - \frac{1}{12}x^6 + C$
8. $\frac{(\ln 5)5^x \sin x - 5^x \cos x}{(\ln 5)^2 + 1} + C$
9. $-x\csc x + \ln|\csc x - \cot x| + C$
10. $\frac{1}{2}x\sqrt{5+x^2} - \frac{5}{2}\ln\left|x + \sqrt{5+x^2}\right| + C$
11. $\frac{x}{81(4x^2-9)^{1/2}} - \frac{4x^3}{243(4x^2-9)^{3/2}} + C = \frac{8x^3 - 27x}{243(4x^2-9)^{3/2}} + C$
12. $\frac{1}{8}\theta - \frac{1}{32}\sin 4\theta + C$
13. $2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$
14. $\frac{3}{2}\sin^{-1}(x-3) + \left(\frac{1}{3}x^2 - \frac{1}{2}x - \frac{11}{6}\right)\sqrt{-x^2+6x-8} + C$
15. $a. M_6 = 24.090235$, $b. T_6 = 24.178500$, $c. S_6 = 24.119610$. Note: Actual ≈ 24.119661
16. a. $R_n = .7811, M_n = .8632, T_n = .8675, L_n = .9540$, b. the integral is between M_n *and* T_n
17. Estimate $ave\,temp = \frac{1}{24}\int_0^{24} T(x)dx \approx \frac{1}{24}S_{12} = 64.2^\circ$. Note: answers will vary slightly from this, since we may differ when we estimate the temperatures from the graph.